The Economics of OTC markets

Lecture 1

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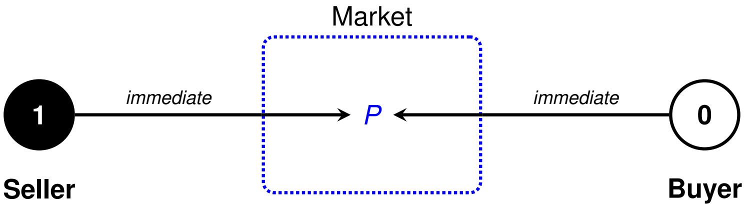
OTC Markets

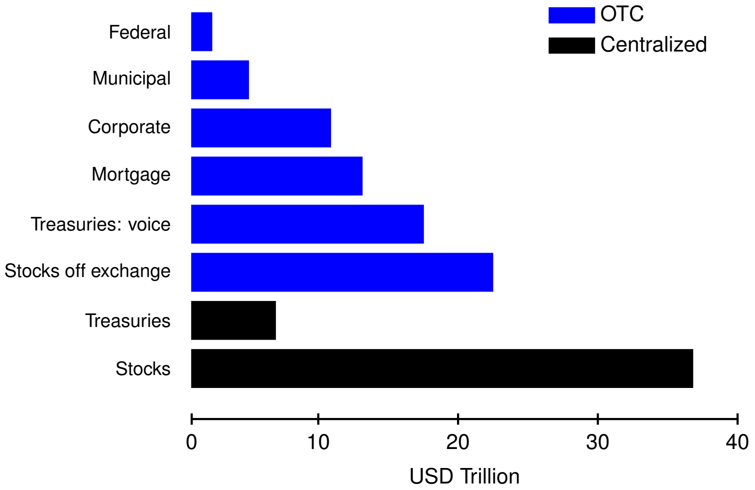
- Trading can be organized in $\neq$ ways
- Decentralized markets
  - Uncertain terms: price, execution delay/cost...
  - Incomplete information
  - Chains
  - Interdealer: Core-periphery
  - Policy: Efficiency? Resilience? Transparency?
- Modelling
  - Search & Bargaining
  - Micro-level dynamic optimization
  - Equilibrium

Centralized trading

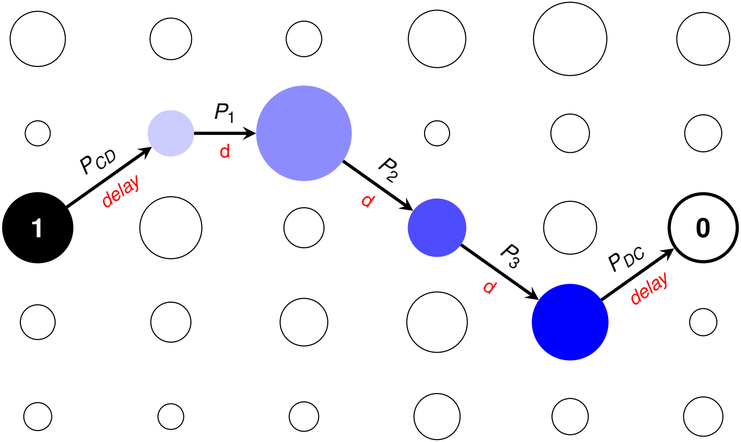


Asset supply (2021)

Excluding derivative securities: OTC $= 1.5 \times$ C

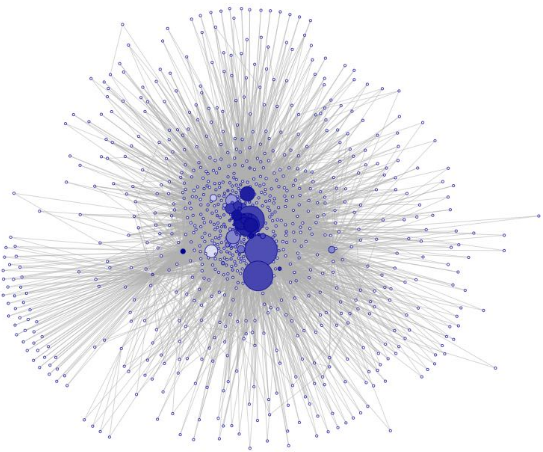


OTC Market



TRACE

Interdealer market for corporate bond



- 1161 dealers in 16/17
- 72 dealers account for 90% of DD trades
- 29 dealers account for 90% of CD and DC trades

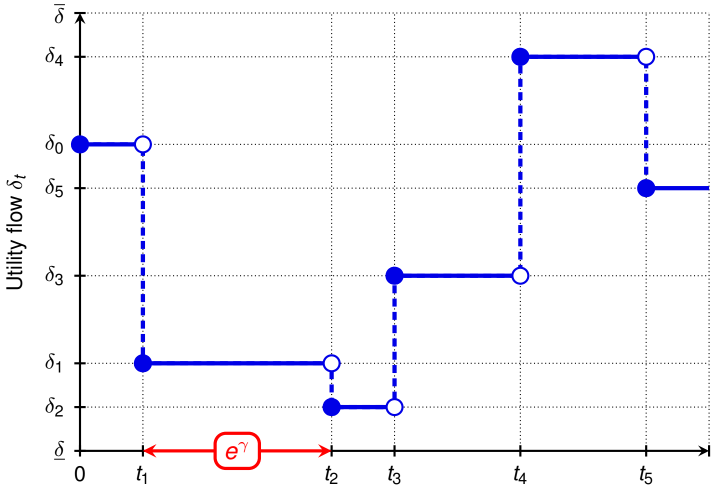
Lectures outline

1. Today
   - Environment
   - Frictionless benchmark
   - Equilibrium with semi-centralized trading
2. Nov.15
   - Multiple dealers: RFQs
   - Equilibrium with decentralized trading
   - Frictional intermediation
3. Nov.29
   - Asymmetric information: Private values
   - Screening investors
   - Directed search

Investors

- **Mass** 1 continuum of investors
  - Infinitely lived with discount rate $r > 0$
  - Can hold $q \in \{0, 1\}$ units of an asset in supply $s < 1$
- Utility **flow** δ_t from holding the asset
  - Jump intensity $\gamma > 0$
  - New value drawn from a CDF F on $\mathcal{D} := (\underline{\delta}, \overline{\delta})$
  - Changes are iid across investors
  - Initial cross-sectional distribution of utility flows is F_0
- Interpretation of utility flows: cost of capital, hedging, consumption opportunities, beliefs or tastes

Utility flow



Formally

- The utility flow (or type) of a generic investor evolves according to a Markov jump process:

$$d\delta_t = \int_{\mathcal{D}} (x - \delta_{t-})\, N(dt, dx)$$

  for some Poisson random measure with

$$\mathbb{E}\left[N(dt, dx)\right] = \gamma dF(x) dt$$

- Each investor has access to the filtration generated by his utility flow and the interdealer market price
- No aggregate uncertainty in the basic model: Equilibrium outputs are constant or depend on time

Demographics

- Denote by $F_t(\delta)$ the *cross-sectional* distribution of utility flows in the population of investors at $t \geq 0$
- By the LLN:

$$\dot{F}_t(\delta) = \text{entry rate}_t - \text{exit rate}_t = \gamma F(\delta) - \gamma F_t(\delta)$$

- Solving this ODE gives

$$F_t(\delta) = F(\delta) + e^{-\gamma t}\left(F_0(\delta) - F(\delta)\right)$$

- Assume from now on that $F_0 = F$ so that $F_t \equiv F$
- This can be relaxed and the model can also accomodate a reset distribution $F = F(\cdot|\delta_{t-})$

Frictionless benchmark

- Continuous trading at price P
- Strategies: adapted processes with values in $\{0, 1\}$
- Objective function

$$Q \longmapsto \mathbb{E}_t \left[\int_t^\infty e^{-r(s-t)} \left(Q_s \delta_s ds - P dQ_s \right) \right]$$
$$= PQ_t + \mathbb{E}_t \left[\int_t^\infty e^{-r(s-t)} Q_s \left(\delta_s - rP \right) ds \right]$$

where the equality follows by integrating by parts on the first line and using the fact that $e^{-rT} Q_T \to 0$

Optimal holdings

- Maximizing gives

$$Q^*(\delta_t) := \underset{q \in \{0,1\}}{\text{argmax}} \{q(\delta_t - rP)\}$$
$$= \mathbf{1}_{\{\delta_t > rP\}} + \mathbf{1}_{\{\delta_t = rP\}} \{0, 1\}$$

- The market clearing condition:

Tie breaking probability

$$s = \int_{\mathcal{D}} Q^*(\delta) dF(\delta) = 1 - F(rP) + \Pi \times \Delta F(rP)$$

implies that $\delta^\star := rP$ is a quantile of the cross-sectional distribution of utility flows at the level $1 - s$

⇒ Frictionless equilibrium is *generically* **unique**

Allocation

- Owners sell upon switching to $\delta' < \delta^\star$
- Nonowners buy upon switching to $\delta' > \delta^\star$
- Investors at the **marginal type** $\delta^\star$ are indifferent
- Equilibrium allocation:

$$\Psi_1(\delta) := \#\{\text{owners}|\delta_t \le \delta\} = (F(\delta) - 1 + s)^+$$

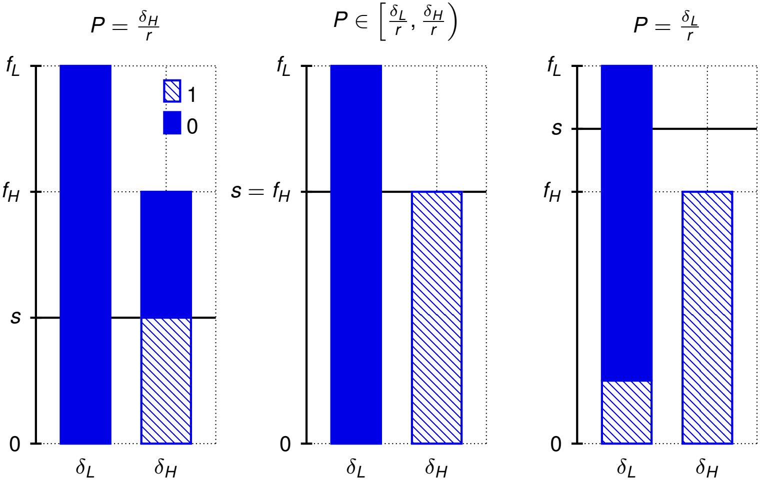
$$\Psi_0(\delta) := \#\{\text{nonowners}|\delta_t \le \delta\}$$

$$= F(\delta) - \Psi_1(\delta) = \max\{1 - s, F(\delta)\}$$

- The allocation is unique even in non generic cases where the marginal type $\delta^\star$ and the price $P = \delta^\star / r$ are not

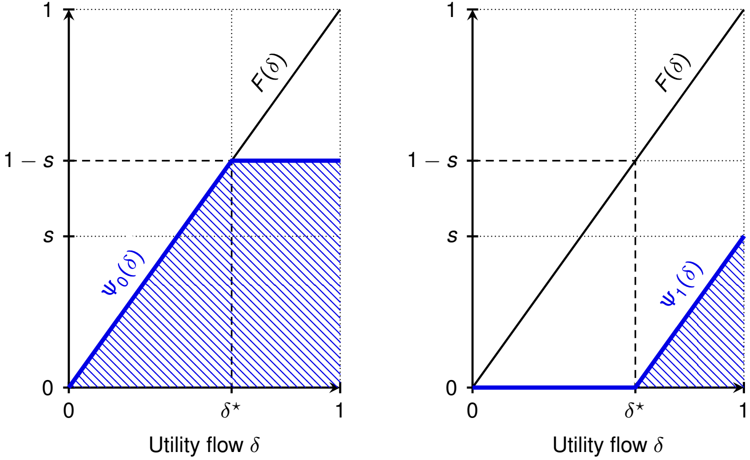
Efficient allocation

Two-points distribution (DGP)



Efficient allocation

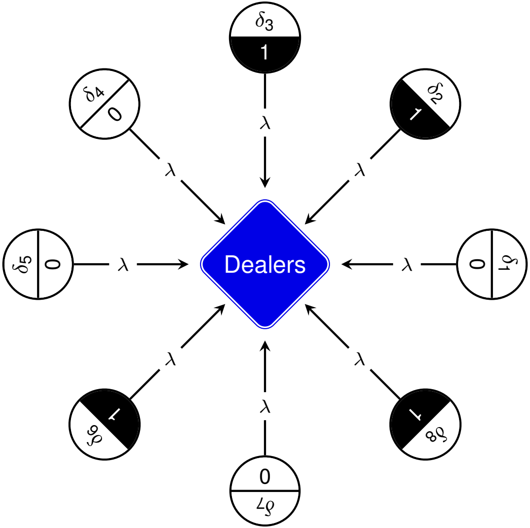
Uniform distribution on the interval $\mathcal{D} = (0, 1)$



Market structure

- Dealers
  - Utility flow $\delta_t \equiv 0$
  - Trade in a frictionless interdealer market
- Investors trade *through* dealers (no CC contacts)
  - Contact with intensity λ
  - Nash bargaining
  - Complete information
  - Bargaining power $\theta \in [0, 1]$ for the dealer
- Frictionless interdealer market: simplification motivated by smaller price dispersion and costs in DD segment

Semicentralized market



Control problem

- Let N_t a Poisson process with rate λ
- A generic investor solves

$$V(q,\delta) := \sup_{a} \mathbb{E}_{q,\delta}\left[\int_0^\infty e^{-rt}\left(Q_t^a \delta_t dt - T\left(Q_{t-}^a, \delta_t\right) dQ_t^a\right)\right]$$

subject to

$$dQ_t^a = a_t\left(1 - 2Q_{t-}^a\right) dN_t \text{ with } Q_0^a = q$$
$$a_t \in \{0,1\}$$

where $T(q,\delta)$ is the amount she pays to or receives from the dealer to execute a trade of size $1-2q$

HJB equation

- Let $\mathbb{E}^F$ denote an expectation relative to F
- The value function satisfies

$$\begin{aligned} rV(q,\delta) &= q\delta + \gamma \mathbb{E}^F \left[V(q,\delta') - V(q,\delta) \right] && \text{(HJB)} \\ \text{(resell)} \qquad & + \lambda q \left(V(0,\delta) - V(1,\delta) + B(\delta) \right)^+ \\ \text{(buy-back)} \qquad & + \lambda (1-q) \left(V(1,\delta) - V(0,\delta) - A(\delta) \right)^+ \end{aligned}$$

where

$$B(\delta) := T(1,\delta) \quad \text{and} \quad A(\delta) := T(0,\delta)$$

denote the bid and ask prices that the investor negociates with the dealer upon meeting

Nash bargaining

- Consider a meeting between a dealer and an owner
- Denote by P the **interdealer** price
- If the surplus

$$V(0,\delta) - V(1,\delta) + P \geq 0$$

then the investor sells at

$$\begin{aligned} B(\delta) &= \underset{b \leq P}{\operatorname{argmax}} \left\{ \left(V(0,\delta) - V(1,\delta) + b \right)^{1-\theta} \left(P - b \right)^{\theta} \right\} \\ &= (1-\theta) P + \theta \left(V(1,\delta) - V(0,\delta) \right) \end{aligned}$$

and retains a share $1-\theta$ of the trade surplus. Otherwise no trade occurs and the investor keeps her asset

Nash bargaining

- A nonowner weakly prefers to buy upon meeting a dealer if and only if her type is such that

$$V(1,\delta) - V(0,\delta) - P \geq 0$$

- In this case she buys at the ask

$$A(\delta) = \underset{a \geq P}{\operatorname{argmax}} \left\{ \left(V(1,\delta) - V(0,\delta) - a \right)^{1-\theta} \left(a - P \right)^{\theta} \right\}$$
$$= (1-\theta) P + \theta \left(V(1,\delta) - V(0,\delta) \right)$$

- The dealer bargaining power θ is fixed
- I will discuss a way to endogenize it in the next lecture

Reservation values

- Denote by

$$R(\delta) := V(1,\delta) - V(0,\delta)$$

the **reservation value** of type δ

- Substituting the bid and ask into (HJB) shows that

$$\begin{aligned} rV(q,\delta) &= q\delta + \gamma\mathbb{E}^F\left[V(q,\delta') - V(q,\delta)\right] \\ \text{(resell)} \qquad &+ \lambda\,(1-\theta)\,q\,(P - R(\delta))^+ \\ \text{(buy-back)} \qquad &+ \lambda\,(1-\theta)\,(1-q)\,(R(\delta) - P)^+ \end{aligned}$$

and subtracting this equation at $q = 0$ from itself at $q = 1$ gives an autonomous equation for reservation values

Reservation values

- RV Equation:

$$rR(\delta) = \delta + \gamma \mathbb{E}^F \left[R(\delta') - R(\delta) \right] + \lambda \left(1 - \theta\right) \left(P - R(\delta)\right)$$

- As-if investors can trade directly in the interdealer market but only at the **reduced** rate $\lambda_\theta := \lambda \left(1 - \theta\right)$
- Because

$$\left(P - R(\delta)\right) = \left(P - R(\delta)\right)^+ - \left(R(\delta) - P\right)^+$$

  we have that the resell (buy-back) option increases (decreases) the reservation value function
- Unique solution has $R'(\delta) = 1 / \left(r + \gamma + \lambda_\theta\right) > 0$

Details

- Subtracting

$$rR(\delta) = \delta + \gamma \mathbb{E}^F \left[R(\delta') - R(\delta)\right] + \lambda_\theta \left(P - R(\delta)\right)$$
$$rR(x) = x + \gamma \mathbb{E}^F \left[R(\delta') - R(x)\right] + \lambda_\theta \left(P - R(x)\right)$$

  shows that we have

$$\left(r + \gamma + \lambda_\theta\right)\left(R(\delta) - R(x)\right) = \delta - x$$

$\Rightarrow$ The solution is unique and Lipschitz continuous

- This is true even if F is discrete
- The solution gives a reservation value to **all types** $\delta \in \mathcal{D}$ not only to existing types $\delta \in \text{supp}F$

Market clearing

- Let $R(\delta^\star) = P$
- Flow of assets brought to the market is λs
- Flow of investors who leave the market with one unit is

$$\lambda\left(1 - F(\delta^\star)\right) + \lambda\Pi \times \Delta F(\delta^\star)$$

where $\Pi \in [0, 1]$ is the fraction of marginal investors who own the asset when parting from a dealer

⇒ Marginal type is a **quantile of F** at the level $1 - s$

- As in the frictionless benchmark!
- Same marginal type $\delta^\star$ but different interdealer price

Interdealer price

- Recall $P = R(\delta^\star)$
- The reservation value equation gives

$$rR(\delta^\star) = \delta^\star + \gamma + \mathbb{E}^F\left[R(\delta') - R(\delta^\star)\right] + \lambda_\theta \overbrace{(P - R(\delta^\star))}^{0}$$

and it follows that

$$rP = \delta^\star + \frac{\gamma}{r + \gamma + \lambda_\theta}\mathbb{E}^F\left[\delta' - \delta^\star\right]$$

- Liquidity **discount**: Interdealer price is lower than in the frictionless benchmark if and only if $\delta^\star > \mathbb{E}^F[\delta']$ so that the marginal investor expects his utility flow to decrease

Allocation

- Denote by $\Phi_{qt} = F - \Phi_{1-qt}$ with $s = \Phi_{1t}(\bar{\delta})$ the distribution of types among investors who hold q assets
- This distribution satisfies

$$\dot{\Phi}_{1t}(\Theta) = \int_{\Theta} \gamma \left(sdF - d\Phi_{1t}\right) + \int_{\Theta} \Big(\lambda_0(x)d\Phi_{0t} - \lambda_1(x)d\Phi_{1t}\Big)$$

where the function

$$\lambda_q(\delta) = \begin{cases} \lambda q, & \text{if } \delta < \delta^\star \\ \in [0, \lambda], & \text{if } \delta = \delta^\star \\ \lambda(1-q), & \text{if } \delta > \delta^\star \end{cases}$$

gives the trading intensity of an investor at (q, δ)

Allocation

- Choosing

$$\Theta(\delta) := \begin{cases} (\underline{\delta}, \delta] & \text{for } \delta < \delta^\star \\ (\delta, \overline{\delta}) & \text{for } \delta \geq \delta^\star \end{cases}$$

allows to show that

$$\dot{\Phi}_{1t}(\delta) = \gamma s F(\delta) - (\gamma + \lambda)\Phi_{1t}(\delta) + \lambda \left(F(\delta) - 1 + s\right)^+$$

- In particular:

$$\lim_{t\to\infty} \Phi_{1t}(\delta) = \frac{\gamma}{\gamma+\lambda} \overbrace{sF(\delta)}^{\text{Random}} + \left(1 - \frac{\gamma}{\gamma+\lambda}\right) \overbrace{\Psi_1(\delta)}^{\text{Efficient}}$$

so that the efficiency of the *steady state* equilibrium asset allocation is entirely controlled by λ/γ

Derivation

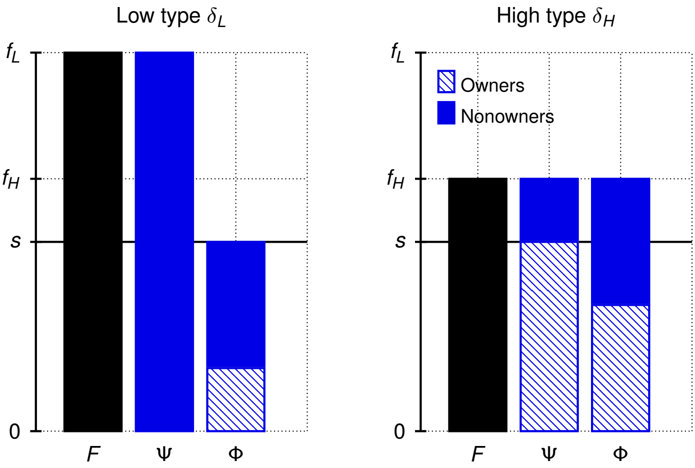
Consider the two cases separately

$$\begin{aligned}\dot{\Phi}_{1t}(\Theta|\delta<\delta^{\star}) &= \dot{\Phi}_{1t}(\delta) \\ &= \int_{\underline{\delta}}^{\delta}\gamma\left(sdF-d\Phi_{1t}\right)-\int_{\underline{\delta}}^{\delta}\lambda d\Phi_{1t} \\ &= \gamma sF(\delta)-(\gamma+\lambda)\,\Phi_{1t}(\delta)\end{aligned}$$

$$\begin{aligned}\dot{\Phi}_{1t}(\Theta|\delta\geq\delta^{\star}) &= -\dot{\Phi}_{1t}(\delta) \\ &= \int_{\delta}^{\overline{\delta}}\gamma\left(sdF-d\Phi_{1}\right)+\int_{\delta}^{\overline{\delta}}\lambda d\Phi_{0} \\ &= (\gamma+\lambda)\Phi_{1t}(\delta)-\gamma sF(\delta)+\lambda\left(1-s-F(\delta)\right)\end{aligned}$$

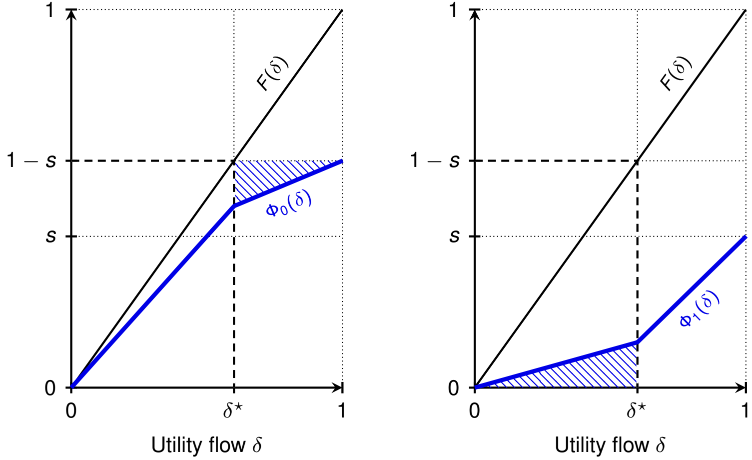
Allocation

Two-points distrbution (DGP)



Allocation

Uniform distribution on the interval $\mathcal{D} = (0, 1)$



Liquidity spread

- The spread is defined by

$$P = \frac{\delta^\star}{r+\ell}$$

and satisfies

$$\frac{\ell}{r+\ell} = \frac{\gamma}{r+\gamma+\lambda_\theta}\left(1 - \mathbb{E}^F\left[\frac{\delta'}{\delta^\star}\right]\right)$$

- The spread is **non zero** even if $\theta = 0$
- The spread is positive iff there is a liquidity discount
- When positive, the liquidity spread decreases with r, λ and increases with γ, θ, and the distress cost $\delta^\star - \mathbb{E}^F[\delta']$

Reservation values

- In equilibrium:

$$rR(\delta) - \delta = \gamma \mathbb{E}^F \left[R(\delta') - R(\delta)\right] + \lambda_\theta \left(P - R(\delta)\right)$$

$$= \gamma \mathbb{E}^F \left[R(\delta') - R(\delta)\right] + \lambda_\theta \left(R(\delta^\star) - R(\delta)\right)$$

$$= (\gamma + \lambda_\theta) \int_{\mathcal{D}} \left(R(\delta') - R(\delta)\right) d\hat{F}(\delta')$$

where the distribution

$$\hat{F}(\delta) := \frac{\gamma}{\gamma + \lambda_\theta} F(\delta) + \left(1 - \frac{\gamma}{\gamma + \lambda_\theta}\right) \mathbf{1}_{\{\delta \geq \delta^\star\}}$$

includes a point mass at the marginal investor and thus is singular relative to any continuous distribution

Reservation values

⇒ It follows that

$$R(\delta) = \mathbb{E}\left[\int_t^{\infty} e^{-r(s-t)}\hat{\delta}_s ds \,\middle|\, \hat{\delta}_t = \delta\right]$$

where the market valuation $\hat{\delta}_t$ follows a Markov jump process with intensity $\gamma + \lambda_\theta$ and reset distribution $\hat{F}$

- Alternatively:

$$R(\delta) = \hat{\mathbb{E}}\left[\int_t^{\infty} e^{-r(s-t)}\delta_s ds \,\middle|\, \delta_t = \delta\right]$$

where $\hat{\mathbf{P}}$ denotes a probability measure under which the type of a generic investor follows a Markov jump process with intensity $\gamma + \lambda_\theta$ and reset distribution $\hat{F}$

Extensions

- Strategic bargaining: $\theta \equiv$ **P** [dlr moves 1st]
- Request for quotes (mini-auction) **L2**
- Relationship trading
- Unrestricted asset holdings
  - Allow $q \in [q_0, \infty)$
  - Asset demand defined by $P = V_q(q^*, \delta)$
  - Marginal value V_q behaves like reservation value R
- Frictional interdealer market **L2**
- Asymmetric information: Private values **L3**
- Directed search **L3**